

Irreducible part of the next-to-leading BFKL kernel

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Abstract

On the basis of previous work by Fadin, Lipatov, and collaborators, and of our group, we extract the "irreducible" part of the next-to-leading (NL) BFKL kernel, we compute its (IR finite) eigenvalue function, and we discuss its implications for small- x structure functions. We find consistent running coupling effects and sizeable NL corrections to the Pomeron intercept and to the gluon anomalous dimension. The qualitative effect of such corrections is to smooth out the small- x rise of structure functions at low values of Q^2 . A more quantitative analysis will be possible after the extraction of some additional, energy-scale dependent contributions to the kernel, which are not treated here.

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The small- x rise of structure functions at HERA [1] has stimulated an impressive theoretical effort [2-15] in order to understand the high-energy behaviour in QCD.

In a series of papers [2-3,5-9,11], Fadin, Lipatov and collaborators have investigated the high-energy cluster expansion (Fig. 1) of the parton-parton cross-section, with the purpose of generalizing the leading-log s BFKL equation[2] to the next-to-leading (NL) order. Similar results for the (heavy) $q\bar{q}$ production cross section[4] and eigenvalues [4, 12], and for the squared gluon emission amplitude [10] have been produced by other authors.

The outcome of such an analysis is the calculation of some "irreducible" vertices which, defined by gluon Regge-pole factorization [2], have the role of incorporating the low energy features of the QCD scattering amplitudes. They are, more precisely, the two-loop gluon trajectory renormalization [8], the one-loop reggeon-reggeon-gluon (RRG) vertex [5-7] and the $RRQ\bar{Q}$ [4] and $RRGG$ [9, 11] clusters at tree-level (Fig. 1 (d-f)).

Such vertices, which involve parton counting in the final state, suffer from mass singularities and need be combined in a sum, with parton number $n = 0, 1, 2$, in order to define the IR finite, irreducible part of the NL kernel. This cancellation of singularities was shown in Ref. [15].

Recently, the N_f -dependent (or $q\bar{q}$) part of the kernel was extracted by the authors [12, 13], its eigenvalue was computed, and the ensuing Pomeron shift and anomalous dimensions were evaluated. Both shift and resummation effects turn out to be suppressed, in this case, by a nonplanar colour factor, which needs not be present in the gluonic case.

Interest in extracting the full gluonic contribution is thus substantial, but is not as simple as in the $q\bar{q}$ case, however. In fact, in order to define the irreducible vertices in the cluster expansion, we need to subtract the leading kernel iteration with $\log s$ accuracy. In addition, we need to define an off-shell scale for the energy, starting from a partonic cross-section which is not an IR safe hard process and thus suffers from Coulomb-like and possibly collinear singularities due to initial partons.

In performing this procedure, we shall distinguish two kinds of contributions to the kernel: (i) the properly irreducible ones, depicted in Fig. 1, which come from the RR ,

RRG and $RRGG$ clusters to be defined below, and (ii) some additional contributions, that we call *energy-scale dependent*, which are remainders of the leading term, with its energy scale, after the required factorization of Coulomb and collinear singularities. The last step involves the choice of a factorization scheme of vertices and kernel which should allow the use of the latter in hard processes.

The purpose of the present note is to perform in part this program, by combining the "irreducible" terms mentioned above and by discussing their eigenvalues and related features. This will allow us to understand the running coupling effects and the main consequences for high energies and anomalous dimension behaviour. On the other hand, full quantitative results can be obtained only after the extraction of the energy scale dependent terms, which is deferred to a subsequent analysis.

Let us start by defining the irreducible terms more precisely. We work in ω -space (Mellin transform in the energy variable s) and transverse momentum space with respect to the incoming partons' axis. The leading kernel in $D = 4 + 2\varepsilon$ dimensions has the well-known [2] form

$$K^{(L)} = \frac{\bar{\alpha}_s}{\omega} K_0(\mathbf{k}_1, \mathbf{k}_2) = \frac{\bar{\alpha}_s}{\omega \Gamma(1 - \varepsilon)} \frac{1}{(\mathbf{k}_1 - \mathbf{k}_2)^2} + \frac{2\omega^{(1)}(\mathbf{k}_1^2)}{\omega} \pi^{1+\varepsilon} (\mu^2)^\varepsilon \delta^{2(1+\varepsilon)}(\mathbf{k}_1 - \mathbf{k}_2), \quad (1)$$

where we have adopted the notation of Fig. 1 (i. e., \mathbf{k} 's (\mathbf{q} 's) for the exchanged (emitted) momenta), and we have introduced the one-loop gluon trajectory [2, 8]

$$\omega^{(1)}(\mathbf{k}^2) = -\frac{\bar{\alpha}_s}{4} C(\varepsilon) \left(\frac{\mathbf{k}^2}{\mu^2} \right)^\varepsilon, \quad \left(C(\varepsilon) \equiv \frac{2}{\varepsilon} \frac{\Gamma^2(1 + \varepsilon)}{\Gamma(1 + 2\varepsilon)} \right), \quad (2)$$

and the notation

$$\bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}, \quad \alpha_s(\mu^2) = \frac{g_\mu^2}{(4\pi)^{(1+\varepsilon)}} \Gamma(1 - \varepsilon). \quad (3)$$

We also understand that transverse integrations carry the measure $d[\mathbf{k}] \equiv d^{2(1+\varepsilon)}\mathbf{k} / \pi^{1+\varepsilon} (\mu^2)^\varepsilon$.

The kernel (1) is the prototype of the ε -dependent kernels to be written out. It is finite for $\varepsilon \rightarrow 0$ and $\mathbf{q} \equiv \mathbf{k}_1 - \mathbf{k}_2 \neq 0$, but may still have singular eigenvalues because

of the $\mathbf{q} = 0$ singularity. The virtual term - in this case the one-loop gluon trajectory - regularizes the singularity by providing a subtraction which yields a finite eigenvalue.

In fact by applying the kernel (4) to the test function $(\mathbf{k}_2^2)^{\gamma-1}$, with $0 < \gamma < 1$, and by using the integral

$$\begin{aligned} \int \frac{d[\mathbf{k}_2]}{\Gamma(1-\varepsilon)\mathbf{q}^2} \left(\frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} \right)^{\gamma-1} &= \left(\frac{\mathbf{k}_1^2}{\mu^2} \right)^\varepsilon \frac{1}{\varepsilon} \frac{\Gamma(1+\varepsilon)\Gamma(\gamma+\varepsilon)\Gamma(1-\gamma-\varepsilon)}{\Gamma(1-\varepsilon)\Gamma(\gamma+2\varepsilon)\Gamma(1-\gamma)} \\ &= \left(\frac{\mathbf{k}_1^2}{\mu^2} \right)^\varepsilon \frac{1}{\varepsilon} \left[\exp(\varepsilon\chi_0(\gamma)) + \frac{1}{2}\varepsilon^2(\psi'(1-\gamma) - 3\psi'(\gamma)) + O(\varepsilon^3) \right] \end{aligned} \quad (4)$$

it is easy to combine real and virtual terms to obtain the characteristic function

$$\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma), \quad (5)$$

where $\psi(\gamma)$ is the logarithmic derivative of the Γ -function.

The NL kernel contains virtual and real emission terms also. The virtual term is the two-loop gluon trajectory

$$K_V^{(NL)} = \frac{2\omega^{(2)}(\mathbf{k}_1^2)}{\omega} \pi(\pi\mu^2)^\varepsilon \delta^{2(1+\varepsilon)}(\mathbf{q}), \quad (6)$$

which, by collecting the gluonic contributions only, is given by [8]

$$\begin{aligned} 2\omega^{(2)}(\mathbf{k}_1^2) &= \frac{\bar{\alpha}_s^2}{4\varepsilon^2} \left[- \left(\frac{\mathbf{k}_1^2}{\mu^2} \right)^\varepsilon \frac{11}{3} \left(1 - \frac{\pi^2}{6}\varepsilon^2 \right) + \right. \\ &\quad \left. \left(\frac{\mathbf{k}_1^2}{\mu^2} \right)^{2\varepsilon} \left(\frac{11}{6} + \left(\frac{\pi^2}{6} - \frac{67}{18} \right) \varepsilon + \left(\frac{202}{27} - \frac{11\pi^2}{18} - \zeta(3) \right) \varepsilon^2 \right) \right]. \end{aligned} \quad (7)$$

The subtraction of the leading term ($\sim 2(\omega^{(1)} \log s)^2$) is in this case unambiguous, because the only available scale is \mathbf{k}_1^2 .

The real emission terms involve several scales, and thus some prescription is required for the subtraction of the leading term. We start by writing the one-gluon emission amplitude with the one-loop corrections of Ref. [5-7] as follows

$$\begin{aligned} M_{\varepsilon a}^{(1)} &= M_{\varepsilon a}^{(0)} \left(1 + \beta_A^{(1)}(\mathbf{k}_1^2) + \beta_B^{(1)}(\mathbf{k}_2^2) + \right. \\ &\quad \left. \frac{1}{2}\omega^{(1)}(\mathbf{k}_1^2) \left(\log \frac{s_1}{\mathbf{q}^2} + \log \frac{s_1}{\sqrt{\mathbf{k}_1^2 \mathbf{k}_2^2}} \right) + \frac{1}{2}\omega^{(1)}(\mathbf{k}_2^2) \left(\log \frac{s_2}{\mathbf{q}^2} + \log \frac{s_2}{\sqrt{\mathbf{k}_1^2 \mathbf{k}_2^2}} \right) \right) + \tilde{M}_{\varepsilon a}^{(1)} \end{aligned} \quad (8)$$

where s_1, s_2 are subenergy variables of the emitted gluon q^μ of polarization ε and colour a , $\beta_A^{(1)}(\beta_B^{(1)})$ are the one-loop corrections [5, 6] to the A (B) vertex, $M^{(0)}$ is the leading amplitude, and $\tilde{M}^{(1)}$ is defined to be the NL irreducible one. They are given by

$$M_{\varepsilon a}^{(0)} = \frac{s}{\mathbf{k}_1^2 \mathbf{k}_2^2} T_a \, 2g_\mu \varepsilon \cdot J, \quad J^\mu(q) = -k_1^\mu - k_2^\mu + \frac{p_A^\mu}{p_A \cdot q}(\mathbf{q}^2 - \mathbf{k}_1^2) - \frac{p_B^\mu}{p_B \cdot q}(\mathbf{q}^2 - \mathbf{k}_2^2), \quad (9)$$

and by

$$\begin{aligned} Re \, \tilde{M}_{\varepsilon a}^{(1)} = & \frac{s}{\mathbf{k}_1^2 \mathbf{k}_2^2} T_a \left[(2g_\mu \varepsilon \cdot J) \frac{\bar{\alpha}_s}{4} \left[-\frac{C(\varepsilon)}{2} \frac{\pi \cos \pi \varepsilon}{\sin \pi \varepsilon} \left(\frac{\mathbf{q}^2}{\mu^2} \right)^\varepsilon + \frac{11}{6\varepsilon} + \right. \right. \\ & + \frac{\mathbf{q}^2}{3} \frac{\mathbf{k}_1^2 + \mathbf{k}_2^2}{(\mathbf{k}_1^2 - \mathbf{k}_2^2)^2} + \left(\frac{11}{6} \frac{\mathbf{k}_1^2 + \mathbf{k}_2^2}{\mathbf{k}_1^2 - \mathbf{k}_2^2} - \frac{2}{3} \mathbf{q}^2 \frac{\mathbf{k}_1^2 \mathbf{k}_2^2}{(\mathbf{k}_1^2 - \mathbf{k}_2^2)^2} \right) \log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \Big] + \\ & + (2g_\mu \varepsilon \cdot J_s) \frac{\bar{\alpha}_s}{4} \left[\frac{\mathbf{k}_1^2 \mathbf{k}_2^2}{3(\mathbf{k}_1^2 - \mathbf{k}_2^2)} \left(11 + \mathbf{q}^2 \frac{2\mathbf{q}^2 - \mathbf{k}_1^2 - \mathbf{k}_2^2}{(\mathbf{k}_1^2 - \mathbf{k}_2^2)^2} \right) \log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} + \right. \\ & \left. \left. + \frac{\mathbf{q}^2}{6} \left(1 - (2\mathbf{q}^2 - \mathbf{k}_1^2 - \mathbf{k}_2^2) \frac{\mathbf{k}_1^2 + \mathbf{k}_2^2}{(\mathbf{k}_1^2 - \mathbf{k}_2^2)^2} \right) \right] \right], \quad (10) \end{aligned}$$

where J^μ is the gluon-emission current associated with high energy scattering [2], while $J_s^\mu = \frac{p_A^\mu}{p_A \cdot q} - \frac{p_B^\mu}{p_B \cdot q}$ is the soft insertion current, occurring in $\tilde{M}^{(1)}$ only.

Notice that we have used in Eq. (10) the small- \mathbf{q}^2 behaviour of Eq. (21) of Ref. [7], together with the fixed \mathbf{q}^2 form of Eq. (86) of Ref. [5]. Furthermore, we have incorporated the $\omega^{(1)} \log \mathbf{k}_i^2$ terms in the definition of the *leading* part in Eq. (8), so as to subtract them out in a scale invariant form.

The one-gluon contribution to the NL kernel comes from the interference of $\tilde{M}^{(1)}$ in Eq. (10) with $M^{(0)}$ in Eq. (9). Using the polarization sums

$$-J^2 = \frac{4\mathbf{k}_1^2 \mathbf{k}_2^2}{\mathbf{q}^2}, \quad J \cdot J_s = \frac{4\mathbf{k}_1 \cdot \mathbf{k}_2}{\mathbf{q}^2}, \quad (11)$$

and performing, for simplicity, an azimuthal average of the polynomial part in \mathbf{q}^2 , we obtain

$$\begin{aligned} K_{1g}^{(NL)} = & \frac{\bar{\alpha}_s}{\omega} \frac{\bar{\alpha}_s}{4} \left[-\frac{C(\varepsilon)}{\Gamma(1-\varepsilon)} \frac{(\mathbf{q}^2/\mu^2)^\varepsilon}{\mathbf{q}^2} \frac{\pi \cos \pi \varepsilon}{\sin \pi \varepsilon} + \right. \\ & \left. + \frac{11}{3} \left(\frac{1}{\varepsilon \Gamma(1-\varepsilon) \mathbf{q}^2} + \frac{1}{\mathbf{k}_1^2 - \mathbf{k}_2^2} \log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \right) \right]. \quad (12) \end{aligned}$$

Finally, the two-gluon emission cluster has been recently computed in Ref [11], where the authors suggest subtracting the leading term

$$\bar{\alpha}_s^2 \int \frac{d[\mathbf{q}_1]}{\mathbf{q}_1^2(\mathbf{q} - \mathbf{q}_1)^2} \int \frac{dx}{x(1-x)} \quad (13)$$

with a scale invariant rapidity phase space $\int \frac{dx}{x(1-x)} = 2 \log(1/\delta)$. By using this prescription, we obtain, from Eq. (20) of Ref. [11], the expression

$$K_{2g}^{(NL)} = \frac{\bar{\alpha}_s}{\omega} \frac{\bar{\alpha}_s}{4} \left[\frac{C(\varepsilon)(\mathbf{q}^2/\mu^2)^\varepsilon}{\Gamma(1-\varepsilon)\mathbf{q}^2} \left[\frac{1}{\varepsilon} - \frac{11}{6} + \left(\frac{67}{18} - \frac{\pi^2}{2} \right) \varepsilon - \left(\frac{202}{27} - 7\zeta(3) \right) \varepsilon^2 \right] + \right. \\ \left. -H_{coll}(\mathbf{k}_1, \mathbf{k}_2) + \tilde{H}(\mathbf{k}_1, \mathbf{k}_2) \right], \quad (14)$$

where we have introduced the "collinear" kernel

$$H_{coll}(\mathbf{k}_1, \mathbf{k}_2) = \frac{1}{32} \left[2 \left(\frac{1}{\mathbf{k}_1^2} + \frac{1}{\mathbf{k}_2^2} \right) + \left(\frac{1}{\mathbf{k}_2^2} - \frac{1}{\mathbf{k}_1^2} \right) \log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} + \left(118 - \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} - \frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} \right) \times \right. \\ \left. \times \frac{1}{\sqrt{\mathbf{k}_1^2 \mathbf{k}_2^2}} \left(\log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \tan^{-1} \frac{|\mathbf{k}_2|}{|\mathbf{k}_1|} + \text{Im } Li_2 \left(i \frac{|\mathbf{k}_2|}{|\mathbf{k}_1|} \right) \right) \right] - \frac{\pi^2}{3\mathbf{k}_>^2}, \quad (15)$$

in which an azimuthal average has been performed, and the dilogarithmic one

$$\tilde{H}(\mathbf{k}_1, \mathbf{k}_2) + \frac{\pi^2}{3\mathbf{k}_>^2} = \frac{2\mathbf{q} \cdot (\mathbf{k}_1 + \mathbf{k}_2)}{\mathbf{q}^2(\mathbf{k}_1 + \mathbf{k}_2)^2} \left[\log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \log \frac{\mathbf{k}_1^2 \mathbf{k}_2^2}{(\mathbf{k}_1^2 + \mathbf{k}_2^2)^2} + \right. \\ \left. + Li_2 \left(1 - \frac{\mathbf{q}^2}{\mathbf{k}_1^2} \right) - Li_2 \left(1 - \frac{\mathbf{q}^2}{\mathbf{k}_2^2} \right) + Li_2 \left(-\frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} \right) - Li_2 \left(-\frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \right) \right] + \\ + 2 \left[\int_0^1 \frac{dt}{(\mathbf{k}_1 - t\mathbf{k}_2)^2} \left(\frac{\mathbf{k}_2 \cdot \mathbf{q}}{\mathbf{q}^2} - \frac{\mathbf{k}_2^2 \mathbf{q} \cdot (\mathbf{k}_1 + \mathbf{k}_2)}{\mathbf{q}^2(\mathbf{k}_1 + \mathbf{k}_2)^2} (1+t) \right) \log \frac{t(1-t)\mathbf{k}_2^2}{\mathbf{k}_1^2(1-t) + \mathbf{q}^2 t} + \right. \\ \left. + (\mathbf{k}_1 \longleftrightarrow -\mathbf{k}_2) \right]. \quad (16)$$

Let us now investigate the physical features emerging from the irreducible NL kernel, as defined by the sum of Eqs. (6), (12) and (14). In order to find its eigenvalue, we shall proceed as for the leading term by applying the kernel to test functions of the form $(\mathbf{k}_2^2)^{\gamma-1}$, with $0 < \gamma < 1$. Due to its explicit renormalization scale dependence we expect the outcome to contain factors of $\log(\mathbf{k}_1^2/\mu^2)$.

We combine first the ε -dependent singular terms in Eqs. (12) and (14) and we notice that the most singular ones ($\sim 1/\varepsilon^2$) cancel out directly,¹ leaving the total real-emission

¹This justifies performing the expansion up to relative order ε^2 , instead of ε^3 .

singular part

$$K_{sing}^{(NL)} = \frac{\bar{\alpha}_s}{\omega} \frac{\bar{\alpha}_s}{4} \left[\frac{(\mathbf{q}^2/\mu^2)^\varepsilon}{\mathbf{q}^2} \frac{C(\varepsilon)}{\Gamma(1-\varepsilon)} \left[-\frac{11}{6} + \left(\frac{67}{18} - \frac{\pi^2}{6} \right) \varepsilon - \left(\frac{202}{27} - 7\zeta(3) \right) \varepsilon^2 + \frac{11}{3} \frac{1}{\varepsilon \mathbf{q}^2 \Gamma(1-\varepsilon)} \right] \right]. \quad (17)$$

This kernel has a finite $\varepsilon \rightarrow 0$ limit at fixed \mathbf{q}^2 , but its eigenvalues are still singular.

We then compute the eigenvalues of the kernel (17) by using Eq. (4) and the additional integral

$$\begin{aligned} \frac{C(\varepsilon)}{\Gamma(1-\varepsilon)} \int \frac{(\mathbf{q}^2/\mu^2)^\varepsilon}{\mathbf{q}^2} d[\mathbf{k}_2] \left(\frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} \right)^{\gamma-1} &= \left(\frac{\mathbf{k}_1^2}{\mu^2} \right)^{2\varepsilon} \frac{1}{\varepsilon^2} \frac{\Gamma^2(1+\varepsilon)\Gamma(\gamma+\varepsilon)\Gamma(1-\gamma-2\varepsilon)}{\Gamma^2(1-\varepsilon)\Gamma(\gamma+3\varepsilon)\Gamma(1-\gamma)} = \\ &= \left(\frac{\mathbf{k}_1^2}{\mu^2} \right)^{2\varepsilon} \frac{1}{\varepsilon^2} \left[\exp(2\varepsilon\chi_0(\gamma)) + \frac{1}{2}\varepsilon^2(4\psi'(1-\gamma) - 8\psi'(\gamma)) + O(\varepsilon^3) \right], \end{aligned} \quad (18)$$

and, by combining them with the virtual term (6) we obtain the finite result

$$\frac{\bar{\alpha}_s}{\omega} \frac{\bar{\alpha}_s}{4} \left[\chi_0(\gamma) \left(-\frac{11}{3} \log \frac{\mathbf{k}_1^2}{\mu^2} + \frac{67}{9} - \frac{\pi^2}{3} \right) - \frac{11}{6} (\chi_0^2(\gamma) + \chi_0'(\gamma)) + 6\zeta(3) \right]. \quad (19)$$

Note now that the coefficient of the $\log \mu$ term is precisely the leading kernel eigenvalue with a beta-function coefficient. Therefore, it can be interpreted as a running coupling factor, much as for the $q\bar{q}$ contribution. We can thus express the total (L+NL) kernel in the form

$$\begin{aligned} K^{(L+NL)} &= \frac{\bar{\alpha}_s(\mu^2)}{\omega} \left[\left(1 - b\alpha_s(\mu^2) \log \frac{\mathbf{k}_1^2}{\mu^2} \right) K_0(\mathbf{k}_1, \mathbf{k}_2) + \alpha_s(\mu^2) K_1(\mathbf{k}_1, \mathbf{k}_2) \right] \\ &\simeq \frac{\bar{\alpha}_s(\mathbf{k}_1^2)}{\omega} (K_0(\mathbf{k}_1, \mathbf{k}_2) + \alpha_s K_1(\mathbf{k}_1, \mathbf{k}_2)), \end{aligned} \quad (20)$$

which defines the NL scale-invariant kernel K_1 .

Factorizing the running coupling at the scale \mathbf{k}_1^2 is an asymmetrical procedure, but is convenient for the discussion of the non-scale-invariant BFKL equation [13]. Using a different scale (e. g., $\alpha_s(\mathbf{k}_>^2)$) implies changing K_1 so as to leave the total NL kernel invariant.

Finally, a straightforward calculation allows the computation of the characteristic function of the remaining finite part of the kernel, except for \tilde{H} , whose eigenvalue is

estimated semi-analytically to be $\tilde{h}(\gamma) \simeq \sum_{n=1}^3 a_n [(\gamma+n)^{-1} + (1-\gamma+n)^{-1}]$, with $a_1 = .72$, $a_2 = .28$, $a_3 = .16$. We thus obtain the gluonic part of the K_1 eigenvalue in the form

$$\begin{aligned} \alpha_s \chi_1^{(g)}(\gamma) = & \frac{\bar{\alpha}_s}{4} \left[-\frac{11}{6} (\chi_0^2(\gamma) + \chi_0'(\gamma)) + \left(\frac{67}{9} - \frac{\pi^2}{3} \right) \chi_0(\gamma) + \right. \\ & \left. + \left(6\zeta(3) + \frac{\pi^2}{3\gamma(1-\gamma)} + \tilde{h}(\gamma) \right) - \left(\frac{\pi}{\sin \pi\gamma} \right)^2 \frac{\cos \pi\gamma}{3(1-2\gamma)} \left(11 + \frac{\gamma(1-\gamma)}{(1+2\gamma)(3-2\gamma)} \right) \right], \end{aligned} \quad (21)$$

to be compared with the N_f -dependent part obtained previously [12]

$$\alpha_s \chi_1^{(q)}(\gamma) = \frac{N_f \alpha_s}{6\pi} \left[\frac{1}{2} (\chi_0^2(\gamma) + \chi_0'(\gamma)) - \frac{5}{3} \chi_0 - \frac{1}{N_c^2} \left(\frac{\pi}{\sin \pi\gamma} \right)^2 \frac{\cos \pi\gamma}{1-2\gamma} \frac{1 + \frac{3}{2}\gamma(1-\gamma)}{(1+2\gamma)(3-2\gamma)} \right]. \quad (22)$$

Correspondingly, the (azimuthal averaged) \mathbf{k} -space kernel can be rewritten in a more compact form:

$$\alpha_s K_1^{(g)} = \frac{\bar{\alpha}_s}{4} \left[-\frac{11}{3\mathbf{q}^2} \log \frac{\mathbf{q}^2}{\mathbf{k}_1^2} \Big|_R - H_{coll}(\mathbf{k}_1, \mathbf{k}_2) + \tilde{H}(\mathbf{k}_1, \mathbf{k}_2) + 6\pi\zeta(3)\delta^{(2)}(\mathbf{q}) \right] \quad (23)$$

where we have used the notation of Eqs. (15) and (16), and we have introduced the regularized distributions in 2-dimensional transverse space

$$f(\mathbf{k}_1, \mathbf{q})|_R = f(\mathbf{k}_1, \mathbf{q}) \Theta(\mathbf{q}^2 - \lambda^2) - \delta^2(\mathbf{q}) \int_{\lambda^2}^{\mathbf{k}_1^2} f(\mathbf{k}_1, \mathbf{q}) d^2\mathbf{q}. \quad (24)$$

The parallel expression for the $q\bar{q}$ -part, obtained in ref [13], has the form²

$$\alpha_s K_1^{(q)} = \frac{N_f \alpha_s}{6\pi} \left[\left(\log \frac{\mathbf{q}^2}{\mathbf{k}_1^2} - \frac{5}{3} \right) \frac{1}{\mathbf{q}^2} \Big|_R - \frac{1}{N_c^2} H_{ab}(\mathbf{k}_1, \mathbf{k}_2) \right], \quad (25)$$

where the abelian contribution is defined by

$$\begin{aligned} H_{ab}(\mathbf{k}_1, \mathbf{k}_2) = & \frac{3}{32} \left[\left(\frac{1}{\mathbf{k}_2^2} - \frac{1}{\mathbf{k}_1^2} \right) \log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} + 2 \left(\frac{1}{\mathbf{k}_1^2} + \frac{1}{\mathbf{k}_2^2} \right) + \right. \\ & \left. + \left(22 - \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} - \frac{\mathbf{k}_2^2}{\mathbf{k}_1^2} \right) \frac{1}{\sqrt{\mathbf{k}_1^2 \mathbf{k}_2^2}} \left(\log \frac{\mathbf{k}_1^2}{\mathbf{k}_2^2} \tan^{-1} \frac{|\mathbf{k}_2|}{|\mathbf{k}_1|} + \text{Im } Li_2 \left(i \frac{|\mathbf{k}_2|}{|\mathbf{k}_1|} \right) \right) \right]. \end{aligned} \quad (26)$$

²Eq. (3.17) of Ref. [13] contains some misprints, in particular a C_f factor instead of N_f .

We notice that in both cases the natural scale of α_s appears to be \mathbf{q}^2 , rather than \mathbf{k}_1^2 , and that the "collinear" and "abelian" terms have same sign and similar structure, but that the latter is suppressed by the $1/N_c^2$ colour factor.

Several comments on the results in Eqs. (20)-(26) are in order. Consider first using them to describe the gluon density in a two-scale hard process. According to our general arguments [13, 14] the kernel with running coupling is consistent with the renormalization group for $\mathbf{k}_1^2 > \mathbf{k}_2^2 \gg \Lambda^2$.

More precisely the Mellin transform $G_\omega(Q, Q_0)$ of the BFKL Green's function is represented by

$$G_\omega(Q, Q_0) = \frac{1}{\gamma_+ \sqrt{-\chi'(\gamma_+)}} \left(\exp \int_{t_0}^t \gamma_+(\alpha_s(t')) dt' \right) K(\omega, t_0) \quad (27)$$

in the anomalous dimension regime

$$t = \log \frac{Q^2}{\Lambda^2} \gg t_0, \quad b\omega t > \chi\left(\frac{1}{2}\right), \quad (28)$$

where $\gamma_+ \simeq \gamma_{gg} + \frac{C_F}{C_A} \gamma_{qg}$ is the larger eigenvalue of the anomalous dimension matrix, defined at NL level by

$$1 = \frac{\bar{\alpha}_s(t)}{\omega} (\chi_0(\gamma_+) + \alpha_s \chi_1(\gamma_+)). \quad (29)$$

From the definition (29) both perturbative and resummed expressions of the NL anomalous dimension follow from the formula

$$\gamma_+^{NL}(\alpha_s, \omega) = -\alpha_s \frac{\chi_1\left(\gamma_L\left(\frac{\bar{\alpha}_s}{\omega}\right)\right)}{\chi_0'\left(\gamma_L\left(\frac{\bar{\alpha}_s}{\omega}\right)\right)}, \quad (30)$$

where $\gamma_L(\bar{\alpha}_s/\omega) = \bar{\alpha}_s/\omega + O(\bar{\alpha}_s/\omega)^4$ is the well-known [2] leading gluon anomalous dimension.

Therefore, the low order expansion of γ_+

$$\gamma_+ = \frac{\bar{\alpha}_s}{\omega} + \alpha_s \left(A_1 + A_2 \frac{\bar{\alpha}_s}{\omega} + A_3 \left(\frac{\bar{\alpha}_s}{\omega} \right)^2 + \dots \right) \quad (31)$$

implies the small- γ behaviour of χ_1

$$\chi_1(\gamma) \simeq \frac{A_1}{\gamma^2} + \frac{A_2}{\gamma} + A_3 + O(\gamma), \quad (32)$$

which can be checked on Eqs. (21) and (22) to be consistent with the known [16] expressions in the DIS scheme

$$\alpha_s A_1 = -\frac{11N_c\alpha_s}{12\pi} - \frac{N_f\alpha_s}{6\pi} \frac{1}{N_c^2}, \quad \alpha_s A_2 = -\frac{N_f\alpha_s}{6\pi} \left(\frac{5}{3} + \frac{13}{6N_c^2} \right). \quad (33)$$

For the gluonic part, H_{coll} is responsible for the collinear behaviour, because \tilde{H} vanishes for either $\mathbf{k}_2^2 = 0$ or $\mathbf{k}_1^2 = 0$.

Furthermore, the gluonic eigenvalue provides, through Eq. (30), important resummation effects which are driven by the negative value $A_1^{(g)} = -11N_c/12\pi$ at the $\gamma = 1$ double pole in Eq. (21). This term causes a rapid increase of $\left(-\gamma^2\chi_1^{(g)}(\gamma)\right)$ relative to its value at $\gamma = 0$ (Fig. 2(a)), which reaches a factor of 2.5 at $\gamma = 1/2$. Thus, unlike the $q\bar{q}$ part, the gluonic part is expected to be relevant for scaling violations at HERA.

The second important point concerns the high-energy behaviour expected for the gluon density. We pointed out in Ref. [14] that in the running α_s case, two kinds of critical ω -values occur. One is the singularity of the anomalous dimension expansion occurring in Eq. (29) close to $\gamma = 1/2$, for which we get an α_s -dependent value

$$\begin{aligned} \omega_{\mathbb{P}}(\alpha_s) &= \bar{\alpha}_s \left(\chi_0 \left(\frac{1}{2} \right) + \alpha_s \chi_1 \left(\frac{1}{2} \right) \right) \\ &= \bar{\alpha}_s \chi_0 \left(\frac{1}{2} \right) (1 - a\bar{\alpha}_s). \end{aligned} \quad (34)$$

The other is the true Pomeron, the ω -singularity dominating the high-energy behaviour beyond the anomalous dimension regime. While the former admits the rough estimate (34), the latter turns out to be dependent on the behaviour of α_s close to $\mathbf{k}_2^2 = \Lambda^2$, and thus cannot be really predicted.

If we take the formula (34) as a qualitative estimate, we realize that the NL gluon contributions in Eq. (22) yield a rather large negative shift, namely $a \simeq 3.4$ with our present knowledge (Fig. 2(b)). This would mean that the "Pomeron" intercept is substantially decreased, of the order of $\omega_{\mathbb{P}} \simeq .2$ for $\alpha_s = .15$.

Let us emphasize that this indication cannot be taken yet as a quantitative estimate, because of the scale-dependent contributions to the kernel that we have neglected and

because various cross-checks of the whole approach are still needed. It means, however, that the NL corrections go in the direction of bridging the gap with soft physics, by smoothing out the small- x rise at low values of Q^2 .

If the above magnitude of NL corrections is confirmed, it raises the problem of the slow convergence of resummed perturbation theory at small- x . Fortunately some classes of corrections can be roughly understood at all orders, because they correspond to physical phenomena we already know about.

One class of corrections is due to the collinear behaviour of large- x contributions, which in the cluster expansion approach occur in higher order clusters, and give rise to multiple poles of $\chi(\gamma)$ at $\gamma = 0$ and $\gamma = 1$. Resumming these poles is mandatory [13] to understand the lower eigenvalue of the anomalous dimension, and in general the behaviour of $\chi(\gamma)$ close to $\gamma = 0$. To see the point, we have plotted in Fig. 4 the function

$$2\psi(1) - \psi(\gamma - A_1\alpha_s) - \psi(1 - \gamma - A_1\alpha_s) + \alpha_s \left(\chi_1(\gamma) - A_1 \left(\frac{\pi}{\sin \pi\gamma} \right)^2 \right) \quad (35)$$

which coincides up to NL level with $\chi_0 + \alpha_s\chi_1$, but differs at higher orders, by a resummation of the collinear behaviour.

It is apparent that the effect of resummation is to displace the $\gamma = 0$ singularity to $\gamma = A_1\alpha_s < 0$, as expected, and to reduce the Pomeron shift, by about 15% for $\alpha_s = .15$.

Another class of corrections was noticed long ago by one of us [17] to be due to coherence effects in the soft gluon emission region. The ensuing structure function equation with angular ordering, further investigated by Catani, Fiorani and Marchesini [18] (the CCFM equation), is the basis for the treatment of such effects to all orders.

From the results which are already available [19], it appears that coherence plays a role starting from the constant A_3 in Eq. (32), and thus from the three loop level. It constitutes, therefore, a rather delicate check of the whole approach, because the scale-dependent terms, neglected here, are also expected to contribute.

On the whole, we think that a full understanding of the next-to-leading kernel will put several phenomenological issues on quantitative grounds and will help to bridge the gap with large- x properties, low Q^2 physics, and diffractive phenomena.

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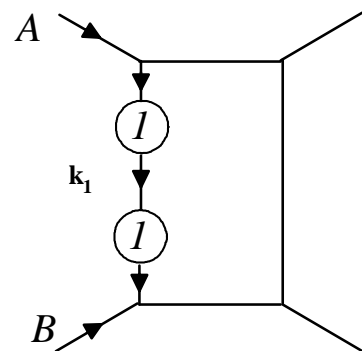
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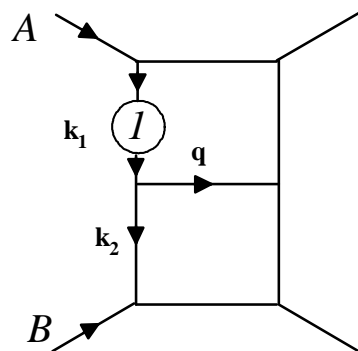
Figure 1: (a-c) Leading and (d-f) next-to-leading contributions to the high energy cluster expansion. The loop number of radiative corrections is specified, and clusters with external particles are omitted. Intermediate quarks and gluons are understood in (f).

Figure 2: Plot of (a) the gluonic contribution to $\gamma^2\chi_1(\gamma)$ and (b) the α_s -dependent "Pomeron".

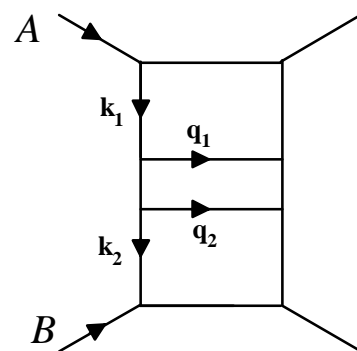
Figure 3: Plot of the resummed characteristic function in Eq. (35), with the symmetrical choice $\chi_1^>(\gamma)$, for which $\alpha_s(\mathbf{k}_>^2)$ is factorized. The corresponding Pomeron shift should be decreased by $\Delta\omega_{\mathbb{P}} = -\frac{b}{2}\alpha_s\bar{\alpha}_s\pi^2$ to compare with the result in Eq. (34).



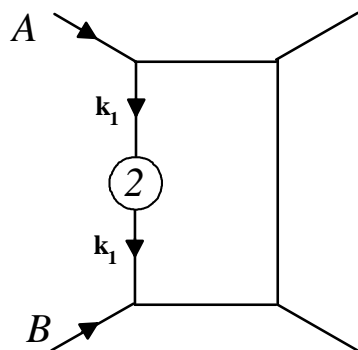
(a)



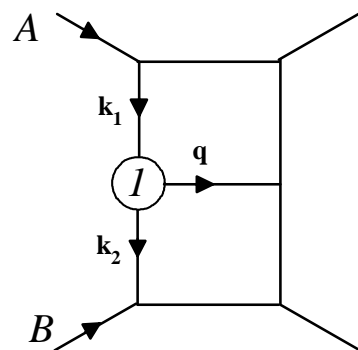
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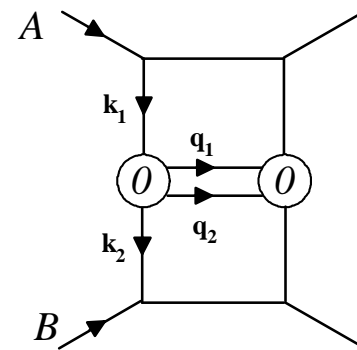
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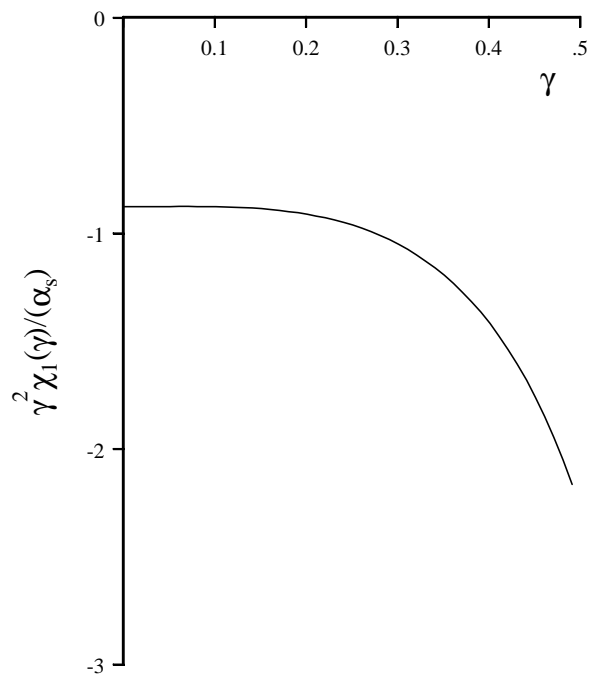
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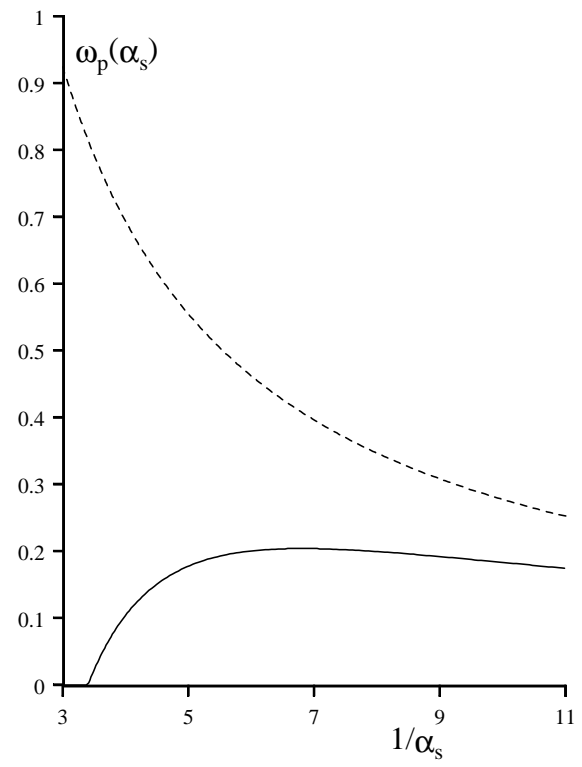
(e)



(f)



(a)



(b)

